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DYNAMICS OF HEAT PRECURSORS IN A MEDIUM CONTAINING CHANNELS WITH ELEVATED THERMAL DIFFUSIVITY

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A heat wave propagating in a locally inhomogeneous medium containing elongated channels with elevated thermal diffusivity is considered.

When shock waves propagate in a locally inhomogeneous medium containing thin elongated channels of lower density, the main shock front becomes distorted and a disturbance of a wedge-like, conic, or other geometry — "a precursor" (see, e.g., survey [1]) – is formed ahead of it. Depending on the strength of the shock wave and the relative rarefaction in the channel the developing two-dimensional or three-dimensional flow may bifurcate, i.e., pass from a regime with a stationary "small-scale" precursor to that with a self-similar growing "large-scale" precursor. At present both regimes are being studied theoretically and experimentally in detail [1].

We will show that the presence of analogous channels with a reduced thermal resistance leads to the appearance of precursors ahead of the fronts of temperature waves as well. It is established that unlike the gasdynamic case, no bifurcation of the process takes place — the heat precursors are always stationary in the limit. Nevertheless, their characteristic sizes may greatly exceed the channel thickness, and the time for establishing a stationary regime of thermal disturbance propagation may be large.

We now illustrate the obtained results by the example of analysis of two problems: problem I on a stationary heat wave and problem II on a "point" thermal explosion.

We consider a simple nonlinear heat conduction model equation in the two-dimensional planar geometry

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\kappa T^n \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\kappa T^n \frac{\partial T}{\partial y}\right) \tag{1}$$

at $n > 0$ in the domain $\{x \geq 0,\ y \geq 0\}$.

The coefficient $\kappa(x, y) = \kappa_0 = \text{const}$ is constant everywhere except for the semi-infinite layer (the plane channel) $\{x \geq \Delta,\ y \leq \delta\}$, in which $\kappa(x, y) = \alpha\kappa_0$, $\alpha \geq 1$.

In problem I, $\Delta = 0$, the initial data are $T(x, y, 0) = 0$, and the boundary conditions are $T(0, y, t) = At^{1/n}$, $\partial T/\partial z(x, 0, t) = 0$.

In problem II, $\Delta = \delta$, the initial temperature in the domain $x^2 + y^2 \leq \delta^2$ is $T(x, y, 0) = T_0$, in the domain $x^2 + y^2 > \delta^2$ it is assumed that $T(x, y, 0) = 0$, and the boundary conditions are

$$\frac{\partial T}{\partial y}(x, 0, t) = \frac{\partial T}{\partial x}(0, y, t) = 0 .$$

In the absence of a disturbing layer (at $\alpha = 1$) both problems are one-dimensional (problem II is in a cylindrical coordinate system) and self-similar. We give, according to [2], an exact solution of problem I that corresponds to a heat wave propagating along the x axis with the constant speed $D = (\kappa_0 A^n/n)^{0.5}$: $T(x, y) = [nD/\kappa_0(Dt - x)]^{1/n}$, $0 \leq x \leq Dt$, $T(x, t) = 0$, $x > Dt$.

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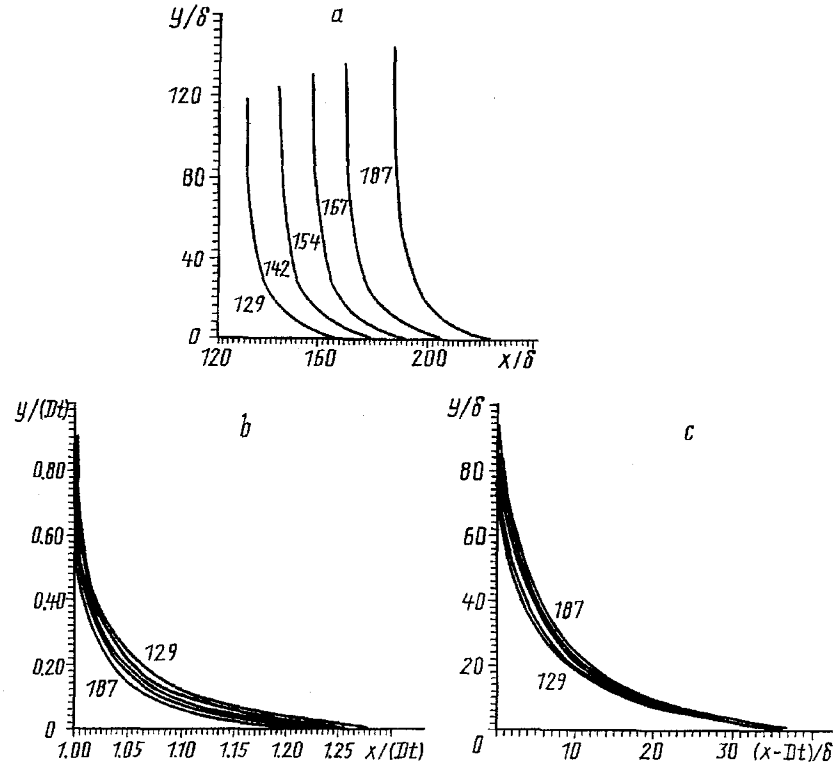

Fig. 1. Stationary heat wave. The form of the temperature front for different moments of time (figures at the curves) in Cartesian (a), self-similar (b), and stationary (c) coordinates.

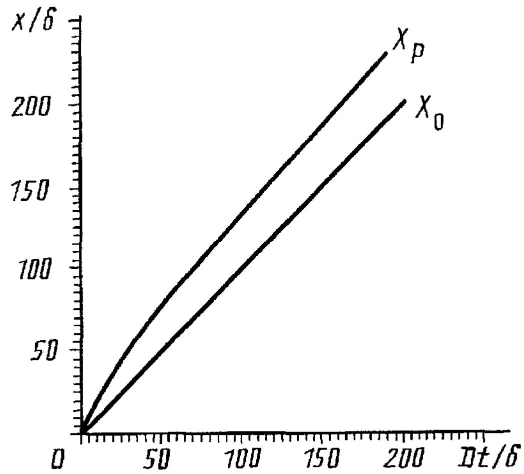

Fig. 2. Stationary heat wave. Coordinates of the fronts of disturbed and undisturbed heat waves as a function of time.

Figure 1 presents a schematic of the front of a two-dimensional heat wave in Cartesian, self-similar, and "stationary" coordinates obtained in numerical calculation of problem I ($n = 2$, $\alpha = 16$) for different moments of time tD/δ (indicated at the corresponding curves). It is seen that the rate of growth of the heat precursor continuously decreases and the configuration of the disturbed temperature front changes from quasi-similar to stationary.

Figure 2 gives the coordinate X_0 of the front of an undisturbed heat wave and the coordinate X_p of the top of the precursor – the front of the disturbed wave – at $y = 0$ as a function of time. There, it is also seen that the solution attains a stationary regime with the limiting length $l \sim 36\delta$ of the precursor.

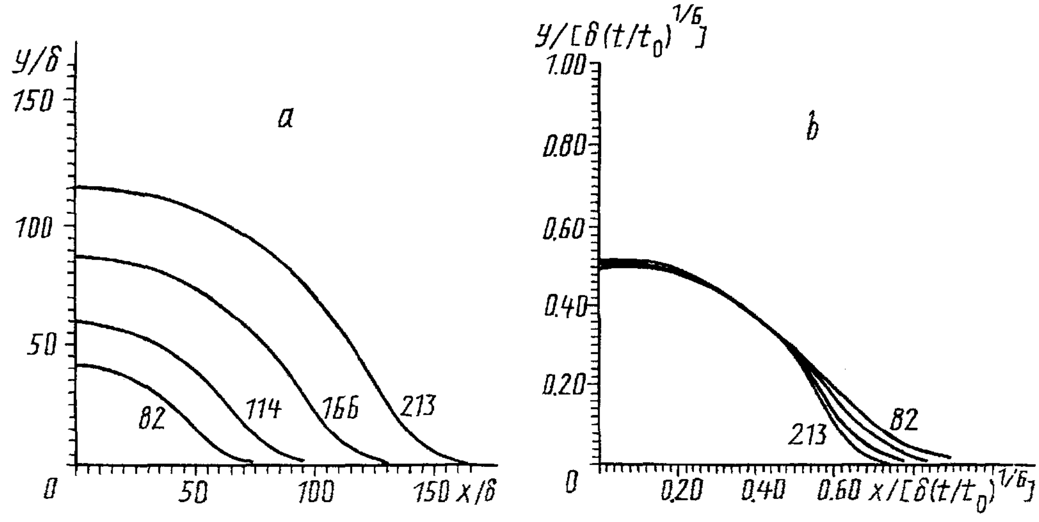

Fig. 3. Thermal explosion. The form of the temperature front at different moments of time (figures at the curves) in Cartesian (a) and self-similar (b) coordinates.

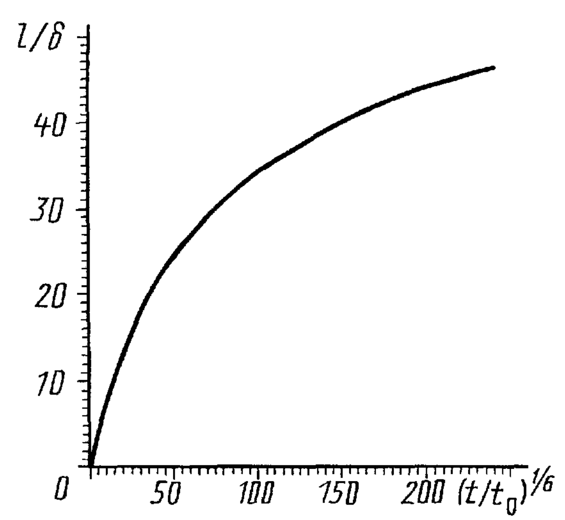

Fig. 4. Thermal explosion. The precursor length versus time.

Using the condition of heat balance in the precursor for the given problem, we may evaluate approximately the length of the precursor at the stationary stage $l = K(\alpha - 1)$, where $K \sim 2-3$. Since α may be much larger than unity, $l >> \delta$.

We now present some calculation results for problem II with the same values of n and α as in problem I. The configuration of the temperature front for different moments of time $(t/t_0)^{1/6}$, indicated at the corresponding curves, where $t_0 = \kappa_0 T_0^2/\delta^2$, in Cartesian and self-similar coordinates is shown in Fig. 3. It is seen that the precursor is "pulled" into a cylindrical wave (in self-similar coordinates) and the configuration of thermal disturbance almost converts into the stationary one (on Cartesian coordinates). In Fig. 4, where the time dependence of $l = X_p - Y_T$ – the difference of the coordinates of the wave fronts along the x and y, respectively, is plotted, the length of the heat precursor is seen to asymptotically attain a stationary value. The limiting value $l \sim 46\delta$, attained in this problem also greatly exceeds the thickness of the channel and is close to the analogous parameter in problem I.

The results obtained show that at sufficiently high values of α the length of the heat precursor may considerably exceed the channel thickness. For instance, in the case of radiative heat conduction in a completely ionized gas $\kappa \sim \rho^{-3}$; when the density in the channel decreases by a factor of 10 compared to the ambient density, α

$\sim 10^3$ and, according to the given estimate, $l/\delta \sim 10^3$. This effect also exists in the case of linear heat conduction (at $n = 0$).

It is pertinent to note that heat precursors in locally inhomogeneous media must also appear in the presence of distributed sources or heat sinks, e.g., ahead of the fronts of combustion waves. It is interesting to verify experimentally the predicted regularities of propagation of heat precursors.

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